Pre-Pilot Studies

Phil Croucher
“Never allow your ego, self-confidence, love of flying, pressure from a customer, boss or co-pilot, or economic need to interfere with your good judgement during any stage of a flight. There is no amount of pride, no thrill, pleasure, schedule or job that is worth your licence or your life and the lives of your passengers. Complacency kills, and so does being a cowboy.” John Bulmer

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INTRODUCTION

This document is a sample of the notes published for the modular self-study programs for the EASA pilot examinations provided by Caledonian Advanced Pilot Training in the UK. It also provides the basic knowledge required before joining the classes. Proper pilot performance is based on knowledge, planning, and anticipation of what the aircraft will do - and you will not be able to achieve that without studying properly. Your real training starts in your first job, and what you learn before then can be very important.

For example, most pilots gain licences from several countries over their careers - if you have a good core knowledge, you will be in and out of the exam rooms a lot quicker. In addition, if you do the minimum work for your exams, by learning the answers rather than the material 😊, it will be painfully obvious to the interview panel when you finally go for a job. Our course go beyond the exams to the technical interview.

“Pure book knowledge should be impeccable - every second of doubt about "what do I do now?" is worth 30% of workload. Mostly because the self-doubt and second-guessing are real time and mental capacity wasters. The more you know flat cold, the easier it is to fly under the gauges”

Nick Lappos

DIFFERENCES

For people coming to the EASA world from North America, some differences are immediately apparent. The maple leaf symbol is meant as a transition aid for Canadian pilots.

First of all, although there are areas where you don't need to speak to anyone on the radio, they are few and far between, and at low level, as almost all airspace is controlled in some way or another (bush pilots take note!) The transition level is also very low, at 3,000 feet in most countries, so get used to those low flight levels.

Next, another barometer setting can be typically used for takeoffs, landings and operations within the circuit, called QFE, which is simply one that gives you a reading of zero feet when on the ground at an aerodrome. It isn't used in North America because many aerodromes are at high elevations and the readings would be off the scale. The setting you are used to, the aerodrome setting against mean sea level, is called QNH.

And what about those Q codes? They are a hangover from wireless telegraphy days, and are not officially supposed to be used, although everyone does (the idea was to use short codes instead of commonly used expressions to reduce transmission times). Flight duty times are shorter, too, and are not part of the exam. You should also join the circuit overhead and there is no UNICOM.
With regard to examinations, it may seem that you are learning a lot of stuff that will not be useful to you. That's certainly true to some extent, but the EASA system makes you learn everything you might need for your career before you start, rather than as you go along - in North America, you will likely be exposed to the same material over the years, but from company ground school and various other type rating courses. It’s just that the Europeans have no guarantee that this will happen and expect you to be a seasoned professional from the start - the original intention behind the EASA exams was to make them the equivalent of a BA degree, since people were regarded as joining a profession. As with many other degrees, a lot of the subject matter was included as padding for credibility purposes, and the main purpose was forgotten. Currently, the EASA ATPL, according to Bristol University, has the same standing as two years of a degree-level course, although the exam procedures are nowhere near as rigorous as that.

However, some of the content is there for third party reasons - Human Factors training is an international requirement, and radio theory must be learnt because you have a cut-down version of the amateur radio licence, and you need to know how not to screw up the airwaves.

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**EQUIPMENT REQUIRED**

For the UK exams, you will also need:

- The **Jeppesen Student Pilot Route Manual**, used in Flight Planning and Navigation. Random copies of this are exchanged at exam time, so don’t write notes on them or highlight anything!

- **Flight computer**. Must be:
  - Jeppesen CR-3 (the one we use)
  - Pooleys CRP-5
  - AFE ARC 2

- **Chart plotting gear**, including a clear ruler marked in mm/cm and inches, 18 ins long at least, dividers, protractor/plotter.
• **Calculator.** Must be:
  • Texas Instrument TI-30XS
  • Sharp EL-W531
  • Citizen SR-260
  • Casio FX-83/85 series
  • Casio FX-300

**Tip:** The hours, minutes, secs functions can save loads of time and avoid costly errors.

The above can be obtained from:
  • Transair (www.transair.co.uk)
  • Pooleys (www.pooleys.com) - mention CAPT for a discount!
  • Airplan Flight Equipment (www.afeonline.com)
  • The Flight & Model Store (www.flightstore.co.uk)

**STUDYING**

It has been found that, within two days, if it isn't reviewed, people remember less than 70% of any subject matter they have studied. By the end of the month, the figure falls to 40%. On the other hand, if it's looked over again within 2 days, then 7, you should be above the 70% level until the 28th day. Another review then should make it remain long-term. In fact, short and frequent bursts of study are more effective than one long one - the brain appears to like short "rests" to assimilate knowledge. **Constant reviewing** is the key, especially for a short time at the end of each day. (Source: *Ohio State University*). In other words, **reinforcement** is necessary for long-term memorisation of any subject matter, an essential component of which is taking notes, especially when the subject matter is not familiar. In any case, the real work is done after the lectures, on your own, which is something that university students know all about.

However, here are a few tips:
  • Allow yourself plenty of time - don’t do everything at the last minute. This means that you need a good routine.
  • If you study during the day, review it in the evening.
  • Get plenty of rest - take some nights off! University students know all about beer, too!

Then you need to practice, practice and practice the exams……..
EXAMS & TECHNIQUE

The clock starts ticking from the month in which you take the first exam, but you can take as long as you want to prepare for it.

- **Rule No 1**: Know your subject!
- **Rule No 2**: Don't take the exams before you're ready - they will be there next time.

The exam questions are multi-choice, with four selections for you to choose from for each question. Although you might use sample ones, or even have access to a database, don't just learn the answers, but read around them (the whole point of these notes), and use variations on them to keep your mind flexible - if you rely on feedback from other people, you need some luck to get the same questions they got - it might help with some subjects, but certainly not Nav Gen or Meteorology. If you know how to do things from the bottom up, or know why things happen, you don't need luck (or a good memory!) You will certainly be a better pilot (would you like to fly with someone who just memorised the answers?)

In the exam room, go through the questions once, and answer those you absolutely and positively know the answer to (this will save a LOT of time!) Do the rest more carefully, looking for where the marks are, remembering that it's entirely possible to get the answer to one question in the text of another, or even some nearly identical, and you will pick them up in the overview. Some questions carry more marks, but will take longer to answer, especially if they involve calculation. If you get stuck, move on and come back later.

Otherwise, there's plenty of time, certainly enough to read each question twice, which sometimes you have to do because the wording is often strange, especially with EASA, where the native language of some examiners is not English (and is often why you get the question wrong). For example, correct numbers may be given in the choices available, but with the wrong units, so read the questions carefully! Most questions have answers that are correct if you make a mistake. Some are worded negatively, such as “What will *not* cause hypoxia?” Also be careful of double-thinking - sometimes the absolute right answer is not available or is not what the examiner wants! In other words, you might not be offered the ideal answer that you already have in your head, but have to choose the best one from a poor selection. Go figure.

Although there's a time limit, nobody cares how quickly you pass, as long as you do, so don't rush, either. Give an answer to every question, even if none of them seem right, so if it turns out to be a bad one, you may get credit for it (if you don't answer it at all, you won't).
Order Of Study

The subjects in our courses are arranged both to provide a logical sequence and to avoid unnecessary duplication, because some subjects share syllabus items. For example, Gen Nav and Instruments share compasses.

Human Performance & Limitations is first because it contains important safety implications that should be taken on board before you start flying or studying (aside from being the one subject that most people at least have some familiarity with). Communications has questions on radio propagation, which is why it comes after Radio Navigation, where it is covered already. Flight Planning is last because it draws on all the other subjects (Performance covers POF as well, and Operational procedures includes some Performance), so you can expect to meet questions on just about anything. By then you should have had plenty of practice at exam technique!

Recommended Reading

The following books and publications (some of which have been used as reference material) are recommended reading for pilots wishing to round out their knowledge:

- Aircraft Instruments & Integrated Systems by E H J Pallett
- Aerodynamics For Naval Aviators (US Govt)
- Weather Forecasting by Tim Vasquez
- Meteorology for Glider Pilots by C E Wallington (out of print)
- Commercial Pilot Studies by Norman Royce (out of print)
- Handbook Of Aviation Meteorology, HMSO
- Air Command Weather Manual, Canadian Government
- Selkirk College Training Materials, Selkirk College BC Canada
- Manual Of Aviation Meteorology (Australian Government)
In many countries (especially Europe), the navigation exam is more to do with maths, using navigation as a background because, to navigate successfully, you need to know how to shuffle numbers and angles around.

Aviation, in common with many other disciplines, uses a precise language so that communication can take place with the minimum of effort. So do maths and science, in the shape of graphs and algebraic symbols. You may also come across circuit diagrams with the electronics involved with radio navigation.

### Numbers

**Factors & Rounding**

Underneath the heading of arithmetic, numbers can be added, subtracted, multiplied or divided (it is assumed that you know how to do them all).

A **prime number** is a natural number (greater than 1) that can only be divided by 1 and itself. A number greater than 1 that is not a prime number is a composite number.

When you divide one number into another, it is a **factor** if the division takes place without leaving a remainder. For example, 4 divides into 20 5 times exactly. If you tried to divide 3 into 20, you would be left with a **remainder** of 2.

Often, if a remainder leaves you some way between two numbers, you must round up or down to get an exact number. If the number is less than halfway, it is the custom to round down, or truncate. If it is halfway and above, you round up.

**The Decimal System**

The following numbers are used in the decimal system:

\[
0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9
\]

Their position determines value. For example, the number 6 has a different meaning in each of the following:

\[
146 \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ 164 \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ 614
\]

In the above examples, it occupies the position of a **unit**, a **ten** or a **hundred**, respectively. A fourth place would be a **thousand**. Thus, the smallest numbers (the units) are always on the right hand side. Zero (0) is often used as a placeholder when a unit is missing:

\[
620 \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ 602
\]
The Sexagesimal System

Time, angles and geographic co-ordinates use a base of 60, as originated by the Sumerians, and handed down to us through the Babylonians and other peoples.

The number 60 has twelve factors, namely 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60, of which 2, 3, and 5 are prime numbers. With so many factors, many fractions involving sexagesimal numbers are simplified. For example, you can divide one hour evenly into sections of 30, 20, 15, 12, 10, 6, 5, 4, 3, 2, and 1 minute(s). 60 is the smallest number that is divisible by every number from 1 to 6; that is, it is the lowest common multiple of 1, 2, 3, 4, 5, and 6.

Positive & Negative Numbers

A number is considered to be positive if it is greater than zero. There is usually nothing in front of positive numbers, but if you mean to make a distinction between positive and negative numbers, you can put a plus sign (+) in front - such as +20°C.

Negative numbers have a minus sign (-) in front of them. You might see this on thermometers when the temperature is colder than freezing (-20°C). However, they are also used in algebra, discussed later.

Fractions

A fraction is a number that is not a whole number, described as vulgar, simple or common fractions. Decimal fractions are discussed below.

Just to confuse matters, a fraction such as ½ is also called a proper fraction, because the numerator (the small number) is above the larger one (denominator). In other words, the fraction has a value of less than 1.

An improper fraction has a value of more than one, such as 22/7, which will become significant as the value of π (pi) which we will come across in Geometry, below.

If you multiply or divide the numerator and denominator by the same number, you get a fraction with the same value as the original one. Reducing a fraction by division is called cancelling. When you can’t cancel any more, the fraction is said to be in its lowest terms.

However, you can only add or subtract fractions that have the same denominator. If you have two with different denominators, you have to find the lowest common denominator, or a number into which they both divide as whole numbers. The lowest common denominator for 3 and 2, for example, is 6. For 4 and 8, it is 8.
For example, with 4 resistances in parallel, of 1, 3, 8 and 15 ohms, to find the unknown total resistance R:

\[
\frac{1}{R} = \frac{1}{1} + \frac{1}{3} + \frac{1}{8} + \frac{1}{15}
\]

The least common denominator is 120, so....

\[
\frac{1}{R} = \frac{120 + 40 + 15 + 8}{120}
\]

which becomes:

\[
\frac{1}{R} = \frac{183}{120}
\]

The non-reciprocal of which is:

\[
R = \frac{120}{183}
\]

As the denominator is greater than the numerator, the answer will be less than 1.

**DECIMAL FRACTIONS**

Decimal fractions work the same way as the numbers do in the decimal system (above), except that the values go from right to left and they are separated from the main number by dot called a decimal point. The figure to the right of the dot represents tenths, the second one hundredths, and so on. For example, 1.5 (one and five tenths) is the same as 1½. 0.01 kilovolts is 10 volts (to multiply a decimal number, simply move the decimal point to the right by the same number of zeros). A recurring decimal (with the same last number multiple times) sometimes has a dot above the last digit, which tells you that it never really divides properly. Although π carries on forever, it is not a recurring decimal but a transcendental number. It’s probably the only one.

**Percentages**

Whereas decimals deal with tens, percentages deal with hundreds, so anything that is a percentage is a part of a hundred. 25% is a fourth part of a hundred, or a quarter.

**Averages**

The word *Mean* (as used in the term *Local Mean Time*) simply means average. Centre of Gravity calculations are averages, where you take a series of numbers, add them up, and divide them by the number of numbers involved. Technically, this gives you an arithmetic mean. The *median* of a set of values is the middle one. The *mode* is the most common value.
MEASUREMENTS

There are two main systems of measurement in general use, metric or Imperial. The metric (or SI) system works in tens, and the Imperial tends to use somewhat arbitrary values (feet, yards, etc.) laid down in the time of Elizabeth I. These include firkins, which are counted in units of two, such as two firkin big, or two firkin heavy, etc.

The SI System is the International System of Units now recommended for scientific purposes instead of CGS (centimetre, gram, and second) and Imperial.

Unfortunately, nobody told the manufacturers, so you will also need a working knowledge of traditional systems, also shown in the tables below.

Primary units are:

<table>
<thead>
<tr>
<th>Item</th>
<th>SI</th>
<th>Anglo-American</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>kilogram (kg)</td>
<td>0.0685 slug</td>
</tr>
<tr>
<td>Weight</td>
<td>newton (N)</td>
<td>0.2248 lbs</td>
</tr>
<tr>
<td>Length</td>
<td>metre (m)</td>
<td>3.281 feet</td>
</tr>
<tr>
<td></td>
<td></td>
<td>39.4 inches</td>
</tr>
<tr>
<td>Time</td>
<td>second (s)</td>
<td>second</td>
</tr>
<tr>
<td>Temperature</td>
<td>kelvin (K)</td>
<td>celsius (C)</td>
</tr>
</tbody>
</table>

These are derived units:

<table>
<thead>
<tr>
<th>Item</th>
<th>SI</th>
<th>Anglo-American</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>kg (9.807 N)</td>
<td>2.2046 lbs</td>
</tr>
<tr>
<td>Density</td>
<td>kg/m³</td>
<td>lbs/ft³</td>
</tr>
<tr>
<td>Pressure</td>
<td>pascal (N/m²)</td>
<td>millibar</td>
</tr>
<tr>
<td>Velocity</td>
<td>m/sec</td>
<td>3.281 ft/sec</td>
</tr>
<tr>
<td>Acceleration</td>
<td>m/s²</td>
<td>3.281 ft/sec²</td>
</tr>
<tr>
<td>g</td>
<td>9.807 m/s²</td>
<td>32.2 ft/sec²</td>
</tr>
<tr>
<td>Power</td>
<td>watt (Nm/s)</td>
<td>.7376 ft.lb/sec hp</td>
</tr>
<tr>
<td>Metric hp</td>
<td>75 kgm/s</td>
<td>.9863 hp</td>
</tr>
<tr>
<td>English hp</td>
<td>76.04 kgm/s</td>
<td>550 ft.lb/sec</td>
</tr>
<tr>
<td>Energy</td>
<td>joule</td>
<td></td>
</tr>
</tbody>
</table>

N/m² is used for wing loading & dynamic pressure.
LENGTH
The basic unit of measurement in the metric system is the metre, as multiplied or divided into kilometres, centimetres, millimetres, etc.

The Imperial system uses inches, feet, yards and miles in that order.

In navigation, a typical length can be expressed as:

- A kilometre, which is 1000 metres, and was originally 1/10,000 of the average distance between the Equator and either Pole on a meridian passing through Paris (thanks to Napoleon, although the Sumerians were there first). It is equivalent to 3280 feet, and 8 km equals 5 statute miles. As a rate, it is expressed in km/hour.

- A nautical mile (nm) is an angular distance taken as an average of 6080 feet, or 1852 m (as a reminder, check out the middle column of your calculator - see right). However, 6076 feet is used by ICAO, so be careful with US calculators (a geographic mile is the distance subtended by one minute at the Equator) A knot is 1 nautical mile per hour. It was originally measured by allowing a rope with a log on the end to stretch out behind a ship. The rope had coloured rags tied in knots at regular intervals, which were counted over time. For aircraft, we need airspeed, groundspeed and relative speed, discussed later.

- A statute mile, which is 5280 feet and is an Imperial measurement, introduced as an arbitrary figure by Queen Elizabeth I. In aviation, it is used only in visibility reports in some countries. 1 nautical mile is equal to 1.15 statute miles.

MASS & VOLUME (CAPACITY)
Units of volume in general use are Imperial Gallons, US Gallons and Litres. Units of mass (weight) are pounds (lbs) and kilograms (kg). To convert from volume to weight and vice versa, you need to know the specific gravity of the liquid concerned, based on that of water, which is taken as 1, since 1 Imperial Gallon of it weighs 10 lbs (1 litre weighs 1 kg). As fuel is less dense than water, a typical SG value (found in most flight manuals) for jet fuel is 0.79, or the equivalent of 7.9 lbs.
ALGEBRA

This is a system of using letters instead of numbers when you are more concerned about the ratio or relationship between objects rather than their values, although algebra can be used to find an unknown value when you know several others.

If you fly 90 miles in 3 hours, your average speed would be 30 knots. This is the result of dividing 90 by 3:

$$\frac{90}{3} = 30$$

The figures would be different for another journey so, to save us writing down different numbers every time, we need a procedure, such as “To find an average speed in knots, divide the number of miles travelled by the number of hours in the air.”

Or, even shorter: “To get an average speed, divide the distance by the time.”

As we are now using more general units, you can use minutes or seconds instead of just hours.

Mathematically, the above could be made even shorter:

$$\text{Average speed} = \frac{\text{Distance}}{\text{Time}}$$

But even that can be tedious, so try:

$$S = \frac{D}{T}$$
or, using the ordinary rules of arithmetic:

$$S = \frac{D}{T}$$

Obviously, you can’t divide letters - they are there to show you what to do with the numbers when you get them. In other words, to use the formula, you substitute the letters for the correct figures. For example, using $x$ to represent almost anything unknown:

$$x + 6 = 8$$

You know that some number plus 6 equals 8. Of course, this is 2. You found that out by subtracting 6 from 8, which is the reverse of addition.

$S$, $D$ and $T$ were chosen above because they suit the problem, but you could have used $A$, $B$ or $C$ or $X$, $Y$ and $Z$, if you remembered their basic meaning.

Other letters can be used when you want to mix different things. You can’t add 3 helicopters and two aeroplanes together, but you can express their relationship like this:

$$2h + 3a$$

Things can also get more complex. If you had two aeroplanes, and you knew the wingspan of one of them, you can find the wingspan of the other without going out in the rain and measuring it (if you had a tape measure long enough) by using a simultaneous equation.

Otherwise, some letters are already reserved, such as $s$ for distance (when Galileo started all this off, he used the word *scale* from his own language). Similarly, Ampere was concerned with the *intensity* of electric current, so he used $I$ to represent it instead of $A$.

**Symbols & Signs**

Because of the limited number of letters in the alphabet, there are also various ways of distinguishing them. For example, if you were faced with several resistors in an electrical circuit, you could label them $R_1$, $R_2$, $R_3$, etc.

However, you should not put the numbers above, like this: $5^2$, $5^3$,... because a number in that position already has a special meaning, such as squaring or cubing, respectively (squaring means multiplying a number by itself, and cubing means doing it three times, and so on).

The number above is an *index*, and it has some curious properties. When you apply an index to the number 10, it is called a power, such as “10 to the power of 2” when you mean $10^2$.

Such powers indicate the number of places the decimal point must be away from 1. It is a convenient way of expressing large (or small) numbers. $10^6$ is also 1,000,000, or 1 with 6 zeros after it. $10^{-6}$ means 0.0000001. $10^{28}$ expresses how many electrons there are in a coulomb, a unit which is used in electricity.
Powers have another property that is made use of in the slide rule part of the flight computer. You can add them together to get the same effect as multiplication - $10^3 + 10^3$ is the same as $10^6$, and you have just multiplied 1 000 by 1 000 to get 1 000 000. When you operate the slide rule, you are not adding numbers, but indices.

Other useful symbols include:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\geq$</td>
<td>Greater than or equal to</td>
</tr>
<tr>
<td>$\leq$</td>
<td>Less than or equal to</td>
</tr>
<tr>
<td>$\approx$</td>
<td>Approximately equal</td>
</tr>
<tr>
<td>$\sqrt{}$</td>
<td>Square root</td>
</tr>
<tr>
<td>$\propto$</td>
<td>Proportionality</td>
</tr>
</tbody>
</table>

Hooke’s Law can use the proportionality sign. For example, the tension in a spring is directly proportional to its extension:

$$T \propto \text{extn}$$

With any such relationship, if one variable is increased by a given factor, such as 2, the other is increased by the same factor, so if you double the tension, you also double the spring’s extension.

**Equations**

An equation is a statement that shows the relationship between quantities, and how they change when one is increased or decreased. You can recognise it by the equals sign (=), and the expressions either side must balance.

To use the Lift Formula as an example, when one of the quantities on the right side is varied, Lift on the left side will follow.

Some letters, when used as symbols, have been allocated meanings by international agreement, or they may change according to the context in which they are used. The Greek symbol $\rho$ (rho) represents air density in Meteorology and resistivity in electronics. Even within disciplines it can change - $\mu$ (mu) can mean amplification factor or permeability, depending on the (electronic) context. When making your own formulae, its best to state the meaning you use to help the person who reads it later.

Say you now knew an average speed and a distance, but needed to find the time. You can just move the figures
around in the formula. What we need to do is get T by itself on one side of the equal sign.

Remembering that the figures either side must balance each other, you can multiply both sides of the equation by the same figure, in this case T. The short cut is just to move T from one side of the equation to the other, and reverse its function:

\[ ST = D \]

Notice that T is now a multiplier, where it was previously a divisor.

Note also that ST is a shorthand way of saying S x T (the period, or full stop, may be used in algebra instead of the multiplication sign - S.T).

If you divided the equation by S you would now get:

\[ \frac{D}{S} = T \]

Which now isolates the time on one side of the equation.

SIMULTANEOUS EQUATIONS

These are used when you have two or more unknown quantities. You need an equation for each one. For example:

\[
\begin{align*}
    a + b &= 8 \\
    a - b &= 4
\end{align*}
\]

The simple way is to cancel the bs out. You end up with:

\[
\begin{align*}
    2a &= 12 \\
    a &= 6
\end{align*}
\]

QUADRATIC EQUATIONS

These involve a square value, of which a positive number will have two - one with a minus value and one with a plus value. A negative number has no roots.

To solve a quadratic equation, turn both sides of it into a square.

As simultaneous and quadratic equations are not often used in aviation, we will proceed to ignore them......
GRAPHS

Pilots use graphs a lot, especially when calculating performance. A graph is a visual representation of the relationship between several numbers, on the basis that it is easier to look at pictures.

The value of zero on whatever scale is called the origin. It comes in useful when drawing tangents from it to the curve of the graph (as used in power graphs).

Slopes

Sometimes you can get information from a graph without looking at any numbers. If the slope of a graph is steep, for example, there is rapid movement. If it is shallow, movement is slow. This is often what happens with temperature readings from the atmosphere.

The value of a straight line graph (where the rise in $y$ is equal to the run of $x$) can be easily calculated with a formula, ending up with $dy/dx$, where $d$ represents a change. However, when the graph is curved, the change in $y$ as compared to $x$ varies, and you need to find some way of calculating the value without drawing the graph. To do this you need to use an infinitesimally small change in order to find the slope of a tangent drawn to that point.

To use another aviation metaphor, if your speed is constant, you can find the distance travelled with a simple calculation (speed x time). However, if your speed is constantly varying, you need something more powerful. You essentially need to calculate the area underneath a sine wave, meaning a rectangle with one side that is reduced and you would have a graphical representation of your fuel use over time. Both sides of the graph need a scale to keep them in proportion to each other.
curved. This kind of calculation (differential calculus) is at the heart of Inertial Navigation/Reference systems. Fortunately, we don’t need to go that far!

EXAMPLE
In meteorology, you can use the position and steepness of a slope to provide information about the weather.

- An environmental (actual) lapse rate (ELR) that follows the DALR creates a condition of neutral stability.
- If the ELR lies to the left of the DALR (being steeper), air is absolutely unstable, which quite rare, except near the ground on sunny days.
- If the lapse rate is between the DALR and SALR (that is, between 1.8 and 3°C per 1000 feet), it is conditionally unstable, meaning stable when dry, but unstable if saturated.
- To the right of the SALR (and the DALR), you get absolute stability from an inversion.

The thick black line is the environmental lapse rate, or that which is occurring on the day, as detected by sending up weather balloons with specialised equipment in them.
Geometry concerns the relationships between lines and angles.

- The **perimeter** is the total length of all the sides of a two-dimensional object. The perimeter of a circle is its circumference.
- The **area** is the space inside the perimeter, found by multiplying the length by the breadth or, in the case of a circle, the square of the radius (half the diameter) multiplied by \( \pi \).
- **Volume** is found by multiplying the area by the height.

**Circles**

These are what makes geometry so interesting. They use the value of \( \pi \), which represents the ratio of the circumference of a circle to its diameter, being 22 divided by 7. It is commonly taken to have a decimal value of 3.1412, but it actually goes on forever.

The **diameter** of a circle is the length of a straight line across it, through the middle. The **radius** goes from the centre to the circumference, or half the diameter.

---

**TRIGONOMETRY**

**Angles & Arcs**

Angles are measured in degrees and radians. A radian is an angle of 57.29º which subtends an arc of the same length as the radius of a circle (it is popular with scientists). 360º = 2\( \pi \) radians. As the radius of a circle is equal to 2\( \pi \), its circumference can be written as the angle in radians round the circle multiplied by the radius (r).

**Note:** Make sure you use the right mode with your electronic calculator!

The logic behind finding the time to a station by measuring the number of degrees you pass through lies with radians. You can see an equilateral triangle inside the circle, except that one side of it is an arc, which sweeps through 57.29º, or 60º for government work (the difference is less than 5% anyway). As the arc is the same length as the radius of the circle, the time taken to fly the arc is the same as it takes to fly to the centre.
If you set up any proportion of distance over time on the flight computer, as in the example below, the speed at which you fly round the arc is shown against the 60 marker, and therefore is the time to get to the station.

For example, your relative bearing to a VOR is 315° which, 3 minutes later, is 270°.

On the flight computer, just set up a ratio of time over degrees passed through (3/45) and look for the answer (4 minutes) against the 60 marker:

4 minutes is the time it would take to fly the straight line between B and C above, although the accurate answer is 3.8 minutes if you look opposite 57.29.

## Triangles

There are three types of triangle:

- An **equilateral** triangle has three sides the same length.

- An **isosceles** triangle has two sides of equal length, and two angles of equal value. The third side is 1.4 times the length of a short side, or the short sides are 70% the length of the long side. As soon as you see a bearing change of 45°, you can bet you are dealing with an isosceles triangle.

- A **scalene** triangle has three sides of different lengths.

All the internal angles of a triangle should add up to 180°. An angle of 90° is called a right angle. One less than that is an **acute angle**. Angles in between are **obtuse** angles.

A right angled triangle has two others that add up to 90°. Pythagoras stated that the square of the hypoteneuse (the long side) is equal to the sum of the squares on the other two sides (although this refers to areas, Pythagoras is usually used to find the length of a side).

When whole numbers are involved, such as 3 – 4 – 5, we have a **Pythagorean Triple**, commonly used in exam questions. For example, in the picture below, the Inertial Navigation System (INS) shows an error on landing – it thinks the aeroplane is in a different place than it actually is, due to certain inbuilt errors, discussed elsewhere.
You can see the proportions involved.

Once you realise that the sides are multiples of 3, 4 & 5, the need for doing any calculations is much reduced!

Two angles that add up to 90° are called co-angles, so angle B is called a cosine (C is already 90°). Its value again depends on dividing the height of the helicopter by distance it has travelled through the air, but the helicopter’s height is now adjacent (i.e. next to) to B, rather than being opposite A.

If you divide the helicopter’s height (opposite) by the distance it has travelled over the ground (adjacent), you have the tangent of angle A.

To find the length of any side, you need to remember these letters:

SOH CAH TOA

The initial letters of each group refer to Sine, Cosine and Tangent, respectively, and the others refer to one side of the triangle, namely Hypoteneuse, Adjacent and Opposite (there is an easy way to remember them below).

Some Old Hens
Can Always Have
Turnips Or Apples

To find Angle A in the previous example, you would therefore use this formula:

\[
\text{Sine} = \frac{\text{Opposite}}{\text{Hypoteneuse}}
\]
To find out which formula you need, take the above letters and cross out the items you know, then use the one where both are crossed out together:

\[
\text{SOH CAH TOA}
\]

Once you have done the division, use your calculator to find the angle corresponding to the result.

These relationships remain the same regardless of the size of the triangles.

**EXAMPLE**

Performance rules require you to clear a building by 35 feet as you get out of a landing site.

You first need to find the angle between the surface and the top of the obstacle. Angle A is 40°.

The tangent of angle A multiplied by the distance to the base of the obstacle gives you the height required, 952 x 0.84 = 800 feet. Then add the 35 feet clearance required (you do it this way because it is easier to line up on the top of the building than try to estimate the height above it).

**Vectors**

A vector is a quantity that has size and direction, such as force or velocity (non-directional scalar quantities like mass have size only, and can be combined by simple addition or subtraction). The length of a vector is proportional to the quantity involved. For example, to represent a speed of 60 knots, you might draw a line 6 inches long, with each inch standing for 10 knots (all the other lines in the drawing must have the same scale).

Now you can work out problems with diagrams rather than complex formulae, because vectors can be combined to produce a resultant such as Total Reaction shown on the right (the single force which is exactly equivalent to two, or more, forces is called their *resultant*). When two forces are applied to or from a point, their resultant is the diagonal of a parallelogram based on that point. The resolution of a vector is the process of finding its effect in two mutually perpendicular directions.

Simple trigonometry (especially Pythagoras) can be used to find the unknown value.
A Vector Diagram is a picture of a vector with an arrow showing the direction the force is acting in. Such a diagram when used in navigation is called the Triangle Of Velocities. Velocity is the rate of change of position in a given direction, equal to distance divided by time. Unfortunately, this is often used synonymously with the word speed, as the units used are the same, but speed is only concerned with the time taken over a distance travelled, not the direction.

**THE 1 IN 60 RULE**

This is a rule of thumb that can solve many problems in aviation without getting the calculator out. The sine or tangent of a small angle is more or less the same as the number of degrees in the angle divided by 60.

Although it is only accurate to within 5% up to about 40° for sines and 10° for tangents, it is a very useful tool (used in Navigation) for quickly working out by how much your track is in error if you have been drifting off.

For example, after flying for 180 miles, you are 9 miles away from your planned track. 9 in 180 is the same as 3 in 60, so you are 3° off (it’s a tangential relationship if you want to work it out properly).
The formula starts off like this:

\[
\text{Error} = \frac{\text{Distance Off}}{60\times\text{Distance Gone}}
\]

It ends up like this:

\[
\text{Error} = \frac{\text{Distance Off} \times 60}{\text{Distance Gone}}
\]

Of course, when you are off track, there is the potential for getting lost, so the first thing to do is parallel the original track. Now, at least, you shouldn’t get any further off track while you work out how to get to the destination.

- **To parallel your original track**, alter course by the track error in the appropriate direction
- **To get back on the original track** (provided you haven’t gone more than halfway), alter course by double the track error. Then apply the correction as a single figure to keep you there
- **To track directly to the original destination**, you would need an extra bit, called a *closing angle*, which you can find by altering the formula above:

\[
\text{CA} = \frac{\text{Distance Off} \times 60}{\text{Distance To Go}}
\]

Add the combination of closing angle and track error to the heading the appropriate way.

**Notes:** The time to regain track may be more than that used to create the error in the first place. Also, these rules are approximate, because altering heading changes the relationship of the wind to your machine. 1 in 60 is used for convenience - if the exact figures for \(\pi\) are used it should be 1 in 57. The Tan may be used up to 25°, and the Sine is accurate up to 40° (within 10% up to 70°).

**Tip:** If you have travelled \(\frac{1}{4}\) of the way along your track, the heading alteration is 4 times the closing angle.

You can use the 1 in 60 rule to see if you are still inside an airway. If the centreline was 045°, and you were on the 040° radial, you would be off track by 5°. If the DME says you are 45 nm away, it’s a simple calculation:

\[
\text{Dist Off} = \frac{\text{TE} \times \text{Dist Gone}}{60}
\]

The answer is 3.75 nm, so you are OK.
Pretty much the whole of aviation works on physics, especially meteorology, so let’s start with the atmosphere.

**ATOMIC THEORY**

Matter is anything that has mass and volume. For our purposes, it exists as a solid, liquid or a gas, but it can be a plasma as well. An element is a substance that cannot be reduced to a simpler form by chemical means because it contains only one type of atom - what distinguishes one element from another is the number of protons, neutrons and electrons in the atoms it contains. A compound contains 2 or more elements - one example is water, which has 2 hydrogen atoms and one of oxygen.

However, for most purposes, the atom is the most basic building block of matter. The word derives from the Greek *a tomos* which means “not cut”, or that you can’t reduce (or cut) the atom into anything smaller, as you can with a molecule, which is a collection of atoms in a chemical compound, the smallest part of an object that retains its identity (meaning that, if you split a molecule, the substance changes its character). By the time Einstein came along, it had been discovered that atoms are both a lot smaller and a lot bigger than was originally thought. If you enlarged an apple until it became the size of the Earth, for example, the atoms inside would be the size of cherries (and the atmosphere would have the thickness of clingfilm). Gold leaf has the thickness of about 5 atoms - if this book were printed on gold leaf, and you multiplied it by four, the total thickness would be that of a single sheet of paper.

The diagram on the left is a loose depiction of the inside of an atom (the Bohr model). The large ball in the middle is the nucleus and the smaller ones spinning rapidly round it are a cloud of electrons, which are negatively charged particles and around 2,000 times smaller in size. The nucleus contains positive- and neutrally charged particles, called protons and neutrons (both contain quarks and other strange things). The neutrons are there to bind the protons together, as particles of a like charge are repelled. As an example of how large atoms can be, if the nucleus were the size of the apple above, the first electron would be found anywhere between 1-10 miles away, and be hardly visible at that.

In an atom, there are an equal number of electrons to protons, to make it electrically neutral, or uncharged. An
atom with one extra electron is *negatively* charged, and an atom with one missing is *positively* charged, or “carrying a positive charge”, which is a bit strange, as all it has done is lost an electron. This is called *ionisation*, because an unbalanced (charged) atom is an *ion*, which we will come across in *Radio Navigation* when we discuss the ionosphere that surrounds the Earth. Some components, like transistors, depend on the movement of electrons or holes (missing electrons) one way or the other.

None of the components of an atom are physical in nature - they are actually electromagnetic charges, or tiny whirlwinds of electromagnetic force. The negative electrons are held in place by the positive protons with *electrostatic attraction*, as particles with opposite charges attract each other. Once an electron leaves an atom, lines of force exist between them, to create a kind of electrical “tension” which is made use of in radio transmissions. Electrons spin round the nucleus at around 600 miles per second so, bearing in mind the relative distances above, you can see that they work quite hard! In fact, they move so quickly round a nucleus that they give the illusion of a more solid construction because our senses don’t work fast enough to detect the difference. So, an atom:

- is not solid
- is mostly full of nothing

Of course, Einstein proved that energy is really matter in another form with his formula:

\[ e = mc^2 \]

In other words, energy is equal to the mass of a body multiplied by the speed of light, squared. Matter converts into energy and back again depending on what you do with its velocity.

**THE ATMOSPHERE**

The atmosphere is an ocean of gases around the Earth, and which moves with it, although it is in continuous motion due to uneven heating.

Various concentric(ish) layers of the atmosphere have been identified over the years. Starting from the bottom, they include the *troposphere*, *stratosphere*, *mesosphere* and *thermosphere*, although the last two are not of much concern to the average pilot. However, the first two layers do concern us, and we live at the bottom of the troposphere, which is at once the thinnest and most dense area because it is compressed by the weight of the air above it. In fact, it contains around 85% of the total mass of the atmosphere. The boundary (or transition zone) between it and the stratosphere is the *tropopause*, where any clouds are made of ice crystals. It lies at an average height of 36 090 feet, or 11 km.
21% of the troposphere, luckily for us, is oxygen, but 78% is nitrogen (N₂), with 1% of odds and ends, like argon (0.9%) and CO₂ (0.03%), and others, that need not concern us here, plus bits of dust and the odd pollutant, and water in various forms in suspension (the nitrogen, as an inert gas, keeps the proportion of oxygen down, since it is actually quite corrosive).

Normally, because of the constant mixing, these proportions remain constant (in dry air*) up to about 80 km, but there are exceptions:

- **Water.** 2% of the Earth’s total water supply can be found suspended in the atmosphere. It would add about an inch of water to the Earth’s surface.
- **Ozone.** 0.001%. *This is toxic*, and the main gaseous constituent of airborne pollution.
- **Carbon Dioxide** (CO₂). 0.05%. This absorbs infrared radiation and allegedly contributes to the greenhouse effect, described in *Meteorology*.

*Not saturated.

If the air wasn't continually being stirred up, the heavier gases would simply sink to the lower levels.

Thus, the atmosphere provides oxygen for us to breathe, and filters out harmful cosmic rays, aside from helping to regulate the Earth’s temperature. The main characteristic of the troposphere is that its temperature falls off with
altitude (because gases cool as they expand), whilst that of the stratosphere is assumed to remain constant until it increases slightly in the latter stages as the Sun’s energy has enough power to heat its molecules directly*. See International Standard Atmosphere, below.

*The ozone layer lies in the middle part of the stratosphere, about 30 kilometres up (between 11-50 km), where the air absorbs ultraviolet radiation from sunlight, to break the bonds of the two atoms that make up oxygen molecules and allow the creation of molecules with three.

Because the lapse rate stops at the Tropopause, and the temperature begins to increase with altitude, the upward movement of air is damped and all the weather is locked into the Troposphere. The Stratosphere is around 30 km thick on average, with its highest and warmest layer at around 50 km above the Earth’s surface. Almost all the remaining 15% of the atmosphere lies within the stratosphere as, above about 25 km, less than 1% remains.

The International Standard Atmosphere

Because the atmosphere (in terms of temperature, pressure and density) changes almost from minute to minute, we need some sort of model to work with, particularly when the volume of a gas changes so much with pressure. You can only get a true idea of the actual quantity of a gas if the volume it would have under some sort of standard is used.

To make sure that everyone works on the same page, a couple of typical scientists went to a typical place (at 40° N latitude) and took the average year round conditions, part of which turned out to be 1013.25 millibars (29.92" of mercury) and 15° Centigrade, which is 288K. This was adopted as the International Standard Atmosphere, and now everyone who makes altimeters, or whatever, calibrates them with it so that everything is standard. In short, ISA is a standard that provides universal values of temperature, pressure, density and lapse rate, by which others can be compared. It not only covers conditions at sea level, but also variations with altitude, although viscosity has not been standardised. The chief difference between actual and standard air is the presence of water vapour, which is more to do with Meteorology. In the standard atmosphere, ½ sea level pressure is obtained at 18 000', one third at 27 500' and ¼ at 33 700'. Thus, pressure decreases with height, but not linearly, because air is compressible and therefore more dense in the lower layers - a layer 1 hectopascal deep is about equal to 27 feet at sea level - at 3 000 feet, it's 30 feet, or around 90 feet at the heights jets fly at, i.e. 35 000 feet. The greatest rate of change is in the lowest 5000 feet. The sea level pressure on which the standard atmosphere is based relates 1" of mercury to 1,000 feet of altitude, so you would expect to see an altimeter read 1 000 feet less if you set it to 28.92 instead of 29.92 inches.
THE GENERAL GAS LAWS

A gas has three variables - pressure (altitude), density and temperature, which are all intimately related. For example, if a gas were restrained in a rigid container (so the volume doesn’t change), increasing the temperature makes the gas expand and increase the pressure inside, and vice versa. If the container were not rigid, the volume could change, and affect the gas’s density. Air density affects aircraft performance. Put another way, you can alter the volume of a gas by changing its pressure or temperature, or both.

Temperature

The quantity of heat contained in a substance is a measure of the kinetic energy of the molecules it contains, depending on the temperature, mass and nature of the material concerned. A bucketful of warm water will melt more ice than a cupful of boiling water because it contains more heat, so two bodies containing the same amounts of thermal energy may not have the same temperature, because temperature is a measure of the quality of heat (or the rate at which molecules are moving), which means it cannot strictly be measured, but only compared against some form of scale.

Officially, temperature is a measure of the average kinetic energy of air molecules measured in Kelvins (K), or absolute temperature (see overleaf).

Two common ways of measuring temperature are Fahrenheit or Celsius, and it’s a real pain to convert between the two. The quick and easy way is to use a flight computer:

But here are the calculations if you want to show off:

\[
\begin{align*}
F - C & \quad Tc = Tf - 32 \times \frac{5}{9} \\
C - F & \quad Tc = \frac{Tf}{\frac{9}{5}} + 32
\end{align*}
\]

They work for any temperatures above freezing. The freezing level (in flight) is where the temperature is 0°C. 16°C is equal to 61°F, 20°C is 68°F and 30°C is 86°F, for gross error checks and quick conversions - however, given the standard of performance charts in the average flight manual, doubling the Celsius figures and adding 30 to get Fahrenheit, or subtracting 30 from Fahrenheit and dividing the remainder in half for Celsius is good enough!

The Fahrenheit scale assumes that water freezes at 32°, and boils at 212° (32° was the coldest possible temperature of an ice-salt mixture. 100° would be the...
temperature of the human body). Centigrade (which is a modified version of the standard introduced by Anders Celsius in the 1700s), starts at 0° and finishes at 100°, which is more logical, but the scale is coarser (the original started at 100°). As the full range of each is 180 and 100 respectively, we get the $\frac{9}{5}$ fraction.

For each °C of cooling, a gas will reduce volume by $\frac{1}{273}$, which brings us into scientific methods of temperature measurement, in the shape of Kelvins, which don’t use a degree sign. -273.15°C is equal to 0 K, or Absolute, which is when all molecular motion is supposed to have stopped, and therefore has the least kinetic energy, although this is scientifically impossible and just used for reference. At this point, there should be zero pressure because the air molecules aren’t moving, which is why the absolute temperature is used in the gas equation.

You could also say that 0°C is equal to 273K, from which you can infer that the 1° steps in both scales are the same.

**Density**

Air density is the mass of air occupying a given volume. It depends on pressure (below), temperature and humidity (water has less mass than air). It is measured in slugs per cubic foot, which are units of mass that accelerate at 1 foot per second when acted on by a force of 1 pound.

**Pressure**

Pressure is the ratio between an applied force that is perpendicular to a surface and the area of the surface concerned. As it is measured in terms of the force it will produce on an area, it should really be expressed in Newtons (sea level pressure is around 101 kN/m²), but, for convenience, we use the (incorrect) values for weight, such as kilograms or pounds.

Static pressure (which plays a major part in breathing, lift, drag, and the operation of carburettors, amongst other things) is proportional to air temperature and density. It arises from the average continuous random
The motion of air molecules. As the random motion involves collisions between them, and they tend to repel each other, the end result is the formation of pressure in all directions. The motions average out at the speed of sound.

Standard atmospheric pressure, or barometric pressure, is the weight of the atmosphere at any given point, at sea level. It depends on the number and mass of air molecules (density), and how fast they are moving (temperature).

At a given height, the only thing that stops the air above you falling to the ground is the pressure of the air below you acting upwards, so the total pressure acting on your aircraft is equal to the weight of the air above it.

The weight of a column of air is commonly expressed in one of three ways:

- **Pounds per square inch.** The force that air exerts in pounds over a square inch of a surface - about 14.7 lbs in the standard atmosphere (2116 lbs per square foot).

- **Inches of mercury.** If you fill a tube with mercury (because it is more dense than water and takes up less space), and tip it upside down into a bowl that is also full of mercury, the level in the tube will drop until the force exerted by atmospheric pressure on the mercury in the bowl equals the weight of the mercury in the tube. Atmospheric pressure under standard conditions will hold up a column of mercury that is 29.92 inches long.

- **Hectopascal.** The Hectopascal (hPa), which is replacing the millibar, consists of 100 Pascals. 1 millibar is equal to 1 Hectopascal.
The weight of the air in a column that is 1 foot square at sea level is 2116.16 lbs (on a standard day). This pressure surrounds an aircraft from above, below and all around. That is, the aircraft is being squeezed from all directions at a static pressure of around 2000 lbs per square foot. If you can reduce the pressure above its aerofoils by more than the weight of the aircraft, it will fly, which is what we do mechanically, by moving forward to concentrate the airflow over the top of the wing and bring its streamlines closer together.

Humidity
To function properly, the human body requires a certain amount of humidity, which concerns the amount of invisible water (vapour) contained in a parcel of air. The absolute humidity is the actual mass, expressed in grams per cubic metre (i.e. as a volume). For a particular temperature, the relative humidity is a measure of how much moisture an air parcel is holding against the maximum it could hold at that temperature (and pressure) or, in other words, the percentage saturation, which will decrease if the air gets warmer. Thus, the amount of water vapour that air can hold is determined by the temperature.

The Ideal Gas
An ideal (perfect) gas obeys the gas laws. As it happens, no gas is really ideal, but they are considered to be so in low subsonic flow, at about 30% of the speed of sound. The kinetic theory of gases (from Maxwell, after Bernoulli) states that gases consist of molecules that are in constant motion, on which their properties depend. The volume of a gas is the space through which its molecules are free to move. From Avogadro’s Law, which states that equal volumes of all gases at the same temperature and pressure contain the same number of molecules (assuming you could count them), you can deduce that the same number of molecules should have the same volume.
Contributions to the kinetic theory of gases include:

- **Charles’ Law**, from a Frenchman, Jacques Charles, which states that, if the pressure remains constant, volume (and density) is very nearly proportional to the absolute temperature so, the hotter a gas gets, the more space it takes up, or the more you compress it into a smaller space, the hotter it gets, and vice versa. If you double the temperature of a gas, you double its volume. Put another way, equal volumes of different gases expand equally for the same temperature if the pressure is kept constant, with the change in volume being $1/273$ of its initial volume at $0^\circ$C, for each degree change in temperature, up or down, so at $-273^\circ$C the volume would be zero. This law (which is only approximately true anyway) helped Charles make the first meteorological flight in a balloon, taking a barometer with which to work out his height. Thus, if Spain and Iceland have the same pressure, the air in Iceland will be denser.

- **Boyle**, an Irish physicist, discovered that, for a perfect gas*, if temperature remains constant (i.e. it is isothermal), its volume (and density) varies inversely with its pressure, so if you double the pressure of a gas, you halve its volume. As you climb, and pressure reduces, the volume of the gases within various body cavities, such as the middle ear, sinuses, the gut, lungs and teeth, increases and may cause pain and/or discomfort.

*Only approximately with high pressures. Boyle’s and Charles’ laws are only accurate in small ranges.

If it’s $25^\circ$C all over Spain, the air density will be lower in the mountains than it is on the beach.

- **Dalton** says that the total pressure of a mixture of gases is the same as the sum of the partial pressures exerted by each of the gases in the mixture, assuming they don’t react chemically with each other, which is relevant for oxygen. In other words, each gas’s pressure contributes a part of the total according to its constituent proportion, or exerts the same pressure that it would do on its own, and the total pressure of the mixture is equal to their sum. This allows meteorologists to figure out how much water vapour there is in a given parcel of air - if they know the makeup of a gas on the ground, they can calculate the amounts for any altitude.

So, after Dalton, if the pressure at a certain altitude were 986 hectopascals, the pressure from oxygen would be 21% of 986, or 207 hPa. An average set of lungs absorbs oxygen at a partial pressure of 3 psi, which is well enough to saturate the blood. The overall and partial pressures of the gases in the atmosphere decrease with increasing altitude.
• Gay-Lussac’s Law states that equal increases in temperature result in equal increases in pressure if the volume is kept constant.

When everything changes at once, you must use Boyle’s and Charles’ laws, in that order. By adding Gay-Lussac and Avogadro to the mix, you can get a single expression called the General Gas Law (also known as the Equation of State), which connects temperature, pressure and density like this:

\[ p = \frac{RT}{\rho} \]

\( \rho \) is the density, \( T \) the absolute temperature and \( p \) the pressure. \( R \) is a constant that depends on the gas (2.87 for dry air). The constant doesn’t change, of course (unless you change the gas), and if temperature stays the same, pressure is proportional to density* - because you are increasing pressure by cramming more molecules into a smaller space, density automatically increases. If pressure stays the same, an increase in temperature reduces the density. So you can calculate density if you know the pressure and temperature.

*If density remains constant, pressure and temperature are directly proportional.

The formula could also arise this way:

\[ PV = RT \]

NEWTON’S LAWS

Aside from inventing the cat flap (true!), Sir Isaac Newton formulated three laws of motion that govern material bodies, which are also relevant to flight (especially 2 & 3 for helicopters):

The First Law

In the absence of an unbalanced force, an object at rest (or in motion) will remain at rest (or in motion at that velocity) until acted upon by an external force, otherwise known as Inertia. Or, if you want the original version:

"Every body perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon"

In other words, an object in a steady state is neither accelerating nor decelerating (although it might be moving) and you must apply a force to make it move or change direction (or if you observe an acceleration, there must be a force behind it). As air has mass, it is capable of applying force, which becomes a problem at the speed of sound because the air compresses so much that it behaves like a brick wall, which is a force in anyone’s language.

Related to the first Law are:
Inertia. A force resisting change that gives a body the tendency to remain at rest, or carry on with what it's doing - in other words, not to change its present state, or to maintain a constant velocity, and be hard to get moving (but see Momentum, below, about stopping). To do its work, air must possess the property of inertia. Inertia should not be confused with Momentum, as even bodies at rest have inertia. Both can add stress to the materials used in aircraft, as found with wings that flex on takeoff or landing. When mass changes, inertia changes, too.

Momentum. The quantity of motion in a body, or its resistance to being brought to rest. As it is a vector quantity, momentum concerns the velocity of a body as well as its mass, so the formula \( \text{mass} \times \text{velocity} \) denotes how much is moving and how fast (only bodies with velocity can have momentum). If either mass or velocity increase, the momentum increases and you need a bigger force to change the body's state of motion. Any change in momentum is proportional to the size of the forces involved - a bullet and a steamroller may have similar momenta, but they have different masses and speeds. For example, a heavy aircraft taxiing at a high rate of knots requires more power to stop than if it were going at walking pace. You could also use a relatively small force for a longer time. The letter \( p \) signifies momentum.

The Second Law

The rate of change of motion (of a body) [acceleration] is directly proportional to the force acting on it, along its line of action, and inversely proportional to the body’s mass. Put more precisely, when a body is made to change its state, its acceleration is proportional to, and in the direction of, the applied force.

Acceleration. The rate of change of motion in speed and/or direction (velocity), divided by time. If you change one or the other, or both, an object is accelerating, as with a turning aircraft affected by centripetal force. Although the word acceleration refers to any change in velocity, deceleration also indicates a decrease.

Force and mass are related, in that doubling both produces the same value of acceleration. Doubling the force doubles the acceleration if mass stays the same, and doubling mass for the same force halves the acceleration, which therefore depends on force divided by mass (and force is proportional to mass multiplied by acceleration). As force is measured in newtons \( (\text{kgm/s}^2) \), represented by the letter \( F \), if 30 newtons is applied to a mass of 10 kg, the acceleration is 30 divided by 10, or 3 m/s\(^2\). 1 kg is equivalent to 9.81 newtons, usually rounded to 10.

A slug is a unit of mass that accelerates at 1 ft/sec when acted on by a force of 1 pound.
The Third Law

If one body exerts a force on another body, the second body will exert an equal and opposite force on the first body, popularised as: For every action, there is an equal and opposite reaction. This law is made use of by propellers and jet engines to drive aeroplanes (and autogyros) forward. In simple terms, the helicopter flies because it pulls down enough air through its rotors to lift it into the air (the Momentum Theory). The tail rotor uses the same principle to stop the fuselage spinning the opposite way to the blades.

Force is a dynamic influence that changes a body’s state of rest to one of motion, or changes its rate of motion. In simple terms, a push (the only forces that truly pull are gravity, magnetism and electrical attraction, and even gravity is suspect these days). It is equal to mass x acceleration (f=m.a). In studying the principles of flight, we are looking at how accelerating a mass (of air) produces a force called Lift that overcomes gravity, or Weight. In this respect, we are interested in its speed and density. The speed concerns kinetic energy, or the additional (dynamic) pressure that is there because the air is moving. However, it is the differences in static pressure that give us lift (and drag), as we shall see later.

In fact, four forces act on an aircraft in flight, called Lift, Weight, Thrust and Drag.

For now, lift makes a flying machine go up, weight makes it go down, thrust makes it go forward, and drag tries to stop it. Creating an imbalance between them is what makes an aircraft go in one direction or another.

- **Centrifugal Force.** Under Newton’s first law, a moving body will travel along a straight path (with constant velocity) unless a force acts on it from the outside. With circular motion, the constant force pushing a body to the centre is centripetal force, inwards along the radius of a curve (attraction). It is an accelerating force, as it affects velocity in terms of its line of direction, and is proportional to the body’s mass.

However, under Newton’s third law the opposite reaction is centrifugal force, or repulsion, which is a fictitious one acting outwards (it is called a reaction force as it is only there because centripetal force is). It increases with mass, the square of rotational speed, and the distance from the axis, as shown:
Centrifugal force is inversely proportional to the radius of the curve, so the smaller a curve is, the more influence centrifugal force has.

Being fictitious, centrifugal force does not act on the body in motion - the only one actually involved is centripetal force. It is the removal of centripetal force that allows a blade to fly from a rotor hub when it is released, not the application of centrifugal force.

- **Couple** is a combination of two equal, parallel and opposite forces that produce a rotation. For example, the couple formed by Thrust and Drag in the picture on the left will cause the nose of the helicopter to go down. It doesn’t go down too far because, once out of line, the new couple formed by lift and weight pulls it back.

- **Moment** is the turning effect of a force about a point, expressed in foot-pounds or newton-metres. **Torque** is similar, but is a continuous force in one direction. The moment of a couple is one of its forces multiplied by the distance between them both (this is relevant for tail rotor drift).

The size of a moment arises from the force involved multiplied by the distance from the point concerned to the line of action of the force.

In the picture below, the lift values are different, yet they balance because of their relative distances from the Centre of Gravity. Clockwise movement is positive, because it involves a nose-up moment, and anticlockwise is negative.

**Equilibrium** is state of balance between forces, where the sum of the clockwise moments is equal to the sum of anticlockwise moments (zero acceleration), as with straight and level flight.

Forces may be in balance, but not in equilibrium - this can happen in a turn with a constant bank angle (where you are accelerating).
Other Definitions

- Axes

  - The **longitudinal axis** of an aeroplane extends fore and aft, through the fuselage. Movement about it is **roll**, controlled by the ailerons, and supplemented by spoilers on larger aircraft, which use two sets of ailerons anyway, one locked out at high speed. This is the only axis with airflow parallel to it, which is relevant for stability. The **angle of bank** (for rolling) lies between the **lateral axis** and the horizon.

  - The **lateral axis** runs from wing tip to wing tip (it parallels the span at 90° to the normal axis). Movement around this axis is **pitch** (the angle between the longitudinal axis and the plane of the horizon) and it is controlled by the elevators.

  - The **vertical** or **normal** axis is perpendicular to the longitudinal and lateral axes. Movement around it is **yaw**, primarily controlled by the rudder, having been moved by the pedals.

All axes run through the Centre of Gravity. **Translation** occurs along an axis, **rotation** occurs about an axis.

- **Velocity.** The rate of change of position in a given direction. Unfortunately, this word is often used instead of **speed**, as the units used are the same, but speed is only concerned with the time taken over a distance travelled, not which way you are going. For example, velocity can be a combination of airspeed and heading, expressing how fast an aircraft is travelling and in which direction.
With respect to velocity:

- **Straight** means flight on a constant heading.
- **Level** means flight at a constant altitude, where the vertical speed is zero.
- **Climbing** means flight at a constant airspeed and constant positive vertical speed (if your airspeed is decreasing you are zooming).
- **Descending** means flight at a constant airspeed and negative vertical speed (if your airspeed is increasing you are diving).
- **Turning** means that your heading is changing. The heading is the angle between the longitudinal axis and a reference line in the plane of the horizon (usually a variety of North).

- **Vector.** A quantity with size and direction, such as force or velocity. Non-directional *scalar* quantities like speed or mass have size only, and can be combined by simple addition or subtraction, whereas the length of a vector is proportional to the quantity involved. For example, a speed of 60 knots might involve a line 6 inches long, with each inch standing for 10 knots (all other lines in the drawing must have the same scale).

Now you can work out problems with diagrams, because vectors can be combined to produce a *resultant* such as Total Reaction shown on the right (the single force which is exactly equivalent to two, or more, forces is called their resultant. When two forces are applied to or from a point, their resultant is the diagonal of a parallelogram based on that point). The *resolution* of a vector is the process of finding its effect in two mutually perpendicular directions. A *vector diagram* is a picture of a vector with an arrow showing the direction the force is acting in.

- **Mass.** The quantity of matter in a body, which is constant if the number of subatomic particles in it remains unaltered. It is functionally identical to weight when gravity is present, as described below. The Centre of Mass of an object is where the sum total of its mass is said to act. The *mass centroid* is a line joining the Centres of Mass of thin slices of a body. It is important when designing propellers and balancing things that rotate. Newton also defined mass in terms of inertia in that, the greater the mass of a body, the greater the force needed to move it.

The basis of flight is the *conservation of mass* (from Lavoisier) applied to a fluid. The principles include:
• **Continuity**, where mass can neither be created nor destroyed, but it may change form into something else, like heat with an engine, or chemical energy (from the engine) into kinetic energy (movement). In a steady flow process (where flow rates don't change over time) the inflow should equal the outflow, or what goes in must come out, whatever might happen in the middle. This is similar to Kirchhoff’s electrical law, and Daniel Bernoulli’s *Venturi Effect*.

• The **Conservation Of Momentum** is Newton’s second law applied to a continuum, but that’s not important right now 😊.

• The **Conservation Of Energy** is similar to the First Law of Thermodynamics (see *Engines* for the second). It is also similar to **Continuity**, in that the energy of a closed system (other things being equal) remains constant during a process.

• **Density** is the mass of a specific volume of air, divided by its volume.

• **Gravity**. The force of attraction between masses, which is greater with mass and closeness together. The **Centre Of Gravity** is the point around which all moments arising from gravity are equal to zero, where an object’s weight (or gravitational attraction) passes through, or where its mass is concentrated (see also *Mass & Balance*). When stationary on the ground, the total weight of an aircraft acts vertically through its Centre of Gravity, parallel to the gravity vector (autopilots rotate aircraft around their Centres of Gravity). The C of G could also be described as the average location of a body’s weight force, or its point of balance. The forward limits are primarily determined by control response and the rear ones by decreasing stability.

• **Weight**. This is the effect of the local gravity vector (g) on a mass that provides a force acting down, toward the centre of the Earth. This may not be constant, as gravity varies around the world but, as the atmosphere occupies only $\frac{1}{600}$ of the space taken up by the Earth, its influence can at least be considered as constant.

  Although pounds and kilograms are commonly used as weight values, they are actually to do with mass - as weight is a
force it should, strictly speaking, be expressed in newtons, which arise from multiplying kg by m/s². \( g \) is shorthand for any force or acceleration that is equivalent to weight, so 2g is twice the weight involved. \( g \) shares the same units as acceleration which, under gravity \((g)\) is 32.2 feet per second, per second, but SI (below) uses metres (9.81 m/s²).

Acceleration as g forces can affect the body in flight (see Human Factors).

- **Work.** A resultant force is said to do work when it moves a body in the direction in which it is acting, so it is equivalent to \( \text{force} \times \text{distance} \), or \( \text{force} \times \text{velocity} \), if you bring time into it, and start thinking in terms of power (below). If an object doesn’t move even if a force is applied, no work is done, although it obviously has in the casual sense.

- **Power.** The rate of doing work, or force \( \times \) velocity, is measured in **horsepower**, which has a standard value of 33,000 ft/lbs per minute, or 550 per second, based on the idea that a standard horse (in a British mine) could lift 100 pounds out of a vertical shaft while walking away at about 4 mph (330 fpm). When you lift a weight, you work against gravity and the power you need depends on the weight of the item concerned and how high you raise it. So, if you lift 10 lbs over 55 feet, or 55 lbs over 10 feet, you require 550 ft-lbs, the product of weight multiplied by distance, but that 550 ft-lbs must be used within 1 second to be a horsepower. The SI unit for power is the watt, which is 1 joule/second. 1 horsepower equates to 0.746 kilowatts, or 746 watts (to convert kW to hp, multiply by 1.34).

- **Pressure.** The force per unit area on a surface arising from the time rate of change of momentum of the gas molecules impacting on it, usually defined at a point normal to the surface.

- **Energy.** A measure of the ability of a body (or unit of mass) to do work, in joules (the unit of work) or newton-metres (weight multiplied by distance). An aircraft can have three types of energy:
  - **Potential energy**, which comes from its position in a gravity field (usually height). 50 lbs at 100 feet has a potential energy of 500 ft-lbs. Similarly, 50 newtons at 100 metres has a potential energy of 5000 joules. When an object is dropped, its potential energy progressively converts to...
  - **Kinetic energy**, which comes from movement (the energy of motion), and is actually a measure of the ability of a body that has velocity to do work when it is brought to rest (mostly a sudden stop!) and can change to pressure energy. If you integrate the rate of change of
momentum with differential calculus, the formula mass x velocity (mv) becomes \( \frac{1}{2}mv^2 \), meaning kinetic energy, which will become significant when we look at Lift.

- **Chemical energy**, from the engines.

An aircraft in straight and level flight has heaps of all three. A reduction in chemical energy (losing an engine) will cause a descent if all other factors remain constant.

- **Viscosity**. Because air has a certain thickness, some of it will stick to an airframe as it tries to push its way through the atmosphere, which takes energy to overcome. This is expressed by viscosity, which is relevant for the internal friction between the layers of oil when it comes to lubrication, and the way air flows over an aerofoil. The higher the viscosity, and the thicker the fluid, the slower the flow.

- **Compressibility**. As soon as you start moving, air is compressed against the frontal surfaces of an aircraft, and may change density. This is ignored below about 300 kts because the molecules repel each other, and the error is only around 5%. Above it, the effects can be significant (see the table below), so any instruments or aerofoils relying on air pressure won’t work so well without adjustment. For any speed above 300 kts, compressibility must be accounted for (300 kts is easily achieved at higher altitudes where TAS increases markedly). The Machmeter automatically corrects for this error.

<table>
<thead>
<tr>
<th>Speed (kts)</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>175</td>
<td>Less than 2%</td>
</tr>
<tr>
<td>260</td>
<td>4%</td>
</tr>
<tr>
<td>347</td>
<td>7%</td>
</tr>
<tr>
<td>436</td>
<td>11%</td>
</tr>
<tr>
<td>522</td>
<td>16%</td>
</tr>
</tbody>
</table>

To predict the effects of compressibility, you need to be able to determine the ...........

- **Mach Number**. Compressibility effects depend on the relationship of airspeed to the speed of sound (see *High Speed Flight*, later). They can be delayed with aerofoil shaping, or streamlining.
• **Angle Of Attack.** The angle between the chord line of an aerofoil and the Relative Airflow.

The most efficient angle of attack at which to fly is where you get the best lift/drag ratio.

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**STREAMLINES & STREAMTUBES**

The passage of air over an aerofoil in a steady path is called a streamline flow*, because a line drawn in a fluid so that its tangent at each point lies in the direction of the fluid’s velocity (at that point) is a streamline.

*When a fluid’s velocity at each point is the same for each particle in terms of time, the motion is described as steady, or a streamline flow. Turbulent flows are unsteady.

Streamlines are therefore imaginary curves along which individual particles of fluid flow. Their density is proportional to their size (or strength) in that, the closer they are together, the stronger is the flow, so they can be used to illustrate increases and decreases in pressure and velocity by being drawn as converging or diverging.

Assuming the air is incompressible, we can say that:
or that the area and velocity on one side of the equation or at one end of the streamtube is the equivalent of those on the other side or at the other end. In other words, if you mess with the area, the velocity changes, and vice versa.

Therefore, if you reduce the cross-sectional area of a tube (or a pipe) and force the streamlines closer together, their velocity will increase (water flows faster if you squeeze the end of a garden hose). Put another way, air flowing into a smaller space must either accelerate or change density because the law of conservation says that matter cannot be destroyed. Below about a third of the speed of sound, the air density will not change, because the natural repulsion between air molecules keeps it more or less constant, but it will do so at more than about 300 knots, which now reduces density, so we are talking about:

\[ \rho_1 A_1 V_1 = \rho_2 A_2 V_2 \]

If you draw a streamline through each point of a closed curve, you get a stream tube, which is a tubular region of fluid surrounded by streamlines. As streamlines do not intersect*, the same streamlines must pass through a streamtube at all points along its length. There is therefore no flow across the surface of a streamtube, so the mass entering it per second is the same as that leaving it per second, as expressed by the continuity equation.